State Probability Analysis of Internet Traffic Sharing in Computer Network

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---**ABSTRACT**---

The Internet is a popular electronic tool to access information around the world. It is controlled by large group of network operators and local Internet service providers. The user is a last node of this network. Billions of people around the world are in the club of internet users at present. The growing demand beyond capability is causing congestion and blocking in networks. There is a kind of inherent competition among operators in market to catch-up more and more users. This paper presents Markov chain model based study of state probability in Internet traffic sharing assuming there are only two operators in a local market in competition. Their network blocking probabilities are mutually compared and simulation study is performed over varying model parameters. It is found that some specific type of users are affected much by networks blocking probability.

Keywords - **Blocking probability, Call-by-call basis, Initial preference, Internet Service Provider [operators], Internet access, Internet traffic, Morkov chain model, Network congestion, Quality of service (QoS), Simulation, Transition probability, Transition probability matrix, Users behavior .**

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I. INTRODUCTION

The facility of Internet is spread out over the world and a large number of people are using this for communication media for their purpose. This fact is leading to a high amount of traffic load on the network due to rigorous generation of calls per second. Additional traffic load constitutes to cause congestion in the flow of information in network. In order to improve their customer base, many operators offer marketing packages to attract on their users. Consumers also desire to have better quality of service from providers. Some users are dedicated to their favourate operators only whereas others are occasion-oriented. Naldi (2002) has discussed a Markov chain model based approach to analyze user's behavior in set up of two operator's environment. This paper provides an extension of this with the addition of one more state in the model. The additional state is an attraction towards the users comfort. Some useful contribution over model based study are due to Naldi (1999) with theories, techniques of Medhi (1991), Perzen (1962), Yuan and Lygeros (2005), Shukla et al.(2007,2009), Shukla and Jain(2007).

II. USER'S BEHAVIOR AND MARKOV CHAIN MODEL

Let ISP_1 and ISP_2 are two Internet service providers in a market. Further, Assume the following for behavior of a user during Internet access:

(i) A user connects his call through either ISP_1 or by ISP_2.
(ii) The user attempts for an ISP, only once and then shift to the next ISP in next attempt and so on. This behavior is termed as call-by-call.
(iii) There are three other options for a user like (a) go for rest (b) abandon the process (c) get success during call connection. The (a) adopts some new marketing plans with probability P_R.
(iv) The initial probability of selection for ISP_1 is p, and for ISP_2 is (1-p).
(v) The blocking probability experienced by the operator ISP_1 is L_1 and by ISP_2 is L_2.

Let $\{Y^{(n)}, n \geq 0\}$be a Markov chain having transitions over the state space {ISP$_1$, ISP$_2$, RS, SS, AP}, where $Y^{(n)}$ denotes the position of Y at the nth attempts ($n \geq 0$), and five states are

State ISP$_1$: first Internet service provider.
State ISP$_2$: second Internet service provider.
State RS : taking rest for a short duration
State SS : success obtained in call connection
State AP : leaving the process for call attempt

Suppose the user is on ISP$_1$ in the nth attempt. If this call blocks with the probability L$_1$ then he may choose either to ISP$_2$ or to RS state in (n + 1)th attempt. User can not be at the same state in two successive call attempts except SS and AP. He can abandon the attempt process at (n + 1)th attempt with probability P$_A$. If reaches to RS from ISP$_2$ in nth attempts then in (n + 1)th attempt he may either with a call on ISP$_1$ with probability r or on ISP$_2$ with probability (1 - r). From RS, user can not move to states SS and AP. The diagrammatic form of transition mechanism in the setup of two Internet service providers is given in fig. 1

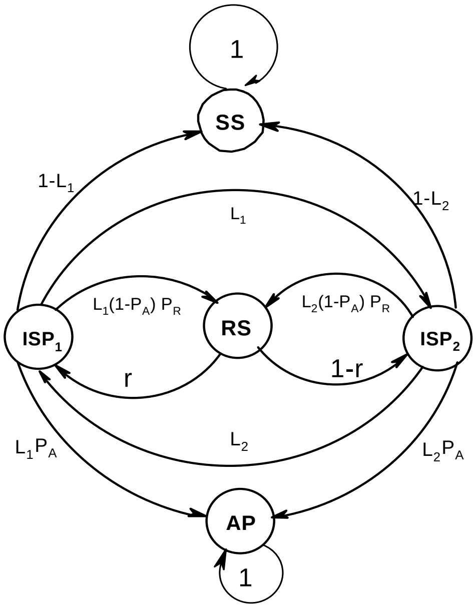

Fig 1 (Transition diagram)

III. TRANSITION PROBABILITIES

(i) The initial probabilities are

$$P\left[Y^{(0)} = ISP_1\right] = p \; ; \; P\left[Y^{(0)} = ISP_2\right] = (1-p) \quad ...(1)$$

(ii) If $(n-1)^{th}$ attempt call for ISP$_1$ is blocked, the user may abandon the process in next attempt.

$$P\left[Y^{(n)} = AP \big/ Y^{(n-1)} = ISP_1\right] = L_1 P_A \quad ...(2)$$

Similar for ISP$_2$, $P\left[Y^{(n)} = AP \big/ Y^{(n-1)} = ISP_2\right]$ = L$_2$ P$_A$

...(3)

(iii) At ISP$_1$ the n^{th} attempt call may be made successful and user reaches to SS from ISP$_1$.

$$P\left[Y^{(n)} = SS \big/ Y^{(n-1)} = ISP_1\right] = \text{P [does not blocked at ISP}_1\text{]}$$

$$= 1\text{-}L_1 \quad ... (4)$$

Similar for ISP$_2$

$$P\left[Y^{(n)} = SS \big/ Y^{(n-1)} = ISP_2\right] = 1\text{-}L_2 \quad ...(5)$$

(iv) At, ISP$_1$ when call is blocked in $(n-1)^{th}$, user does not want to abandon, but wants a little relax then,

$P\left[Y^{(n)} = RS \big/ Y^{(n-1)} = ISP_1\right]$ = P[blocked at ISP$_1$] P[does not abandon] P[wants a little rest] $= L_1(1-P_A)P_R$

... (6)

For ISP$_2$, $P\left[Y^{(n)} = RS \big/ Y^{(n-1)} = ISP_2\right] = L_2(1-P_A)P_R$

...(7)

(v) At ISP$_1$, if user is blocked at ISP$_1$ in $(n-1)^{th}$ and shifts over to ISP$_2$.

$P\left[Y^{(n)} = ISP_2 \big/ Y^{(n-1)} = ISP_1\right]$ = P[blocked at ISP$_1$] P[not abandon] P[not have rest] $= L_1(1-P_A)(1-P_R)$

- - - (8)

For ISP$_2$, $P\left[Y^{(n)} = ISP_1 \big/ Y^{(n-1)} = ISP_2\right] = L_2(1-P_A)(1-P_R)$

- - - (9)

(vi) Also, we could express for, $0 \leq r \leq 1$

$$P\left[Y^{(n)} = ISP_1 \big/ Y^{(n-1)} = RS\right] = r \; ;$$

$$P\left[Y^{(n)} = ISP_2 \big/ Y^{(n-1)} = RS\right] = 1-r \quad ---(10)$$

(vii) The states SS and AP are absorbing states.
The transition probability matrix is in fig 2.

States

$$\begin{array}{cc|ccccc} & & \multicolumn{5}{c}{\longleftarrow \quad Y^{(n)} \quad \longrightarrow} \\ & & ISP_1 & ISP_2 & RS & SS & AP \\ \hline \uparrow & ISP_1 & 0 & L_1(1-P_A)(1-P_R) & L_1(1-P_A)P_R & 1-L_1 & L_1P_A \\ & ISP_2 & L_2(1-P_A)(1-P_R) & 0 & L_2(1-P_A)P_R & 1-L_2 & L_2P_A \\ Y^{(n-1)} & RS & r & 1-r & 0 & 0 & 0 \\ & SS & 0 & 0 & 0 & 1 & 0 \\ \downarrow & AP & 0 & 0 & 0 & 0 & 1 \end{array}$$

- - - (11)

Fig 2 (Transition probability matrix)

IV. QUALITY OF SERVICE (QOS)

The quality of service (QoS) provided by an ISP is a function of blocking probabilities (L_1 and L_2) faced by internet service providers due to congestion in the network. Higher level of blocking probability leads to lesser quality received by users. As per assumptions of the system, a user is suppose to attempt for calls between ISP_1 and ISP_2 until connects or may take rest if fed-up due to attempt process.

V. USER'S CATEGORIZATION

Based on position of system in n attempts, one gets:

(a) Faithful User (FU):

Who is faithful to ISP_1 otherwise prefer for the rest state RS or abandon but does not attempt for ISP_2. The converse of same is for ISP_2. A group of this kind is defined as faithful users for ISP_1 {or ISP_2.}.

(b) Partially Impatient User (PIU):

Who attempts only between the two service providers ISP_1 and ISP_2, all the time until call completes or abandon but never goes to RS.

(c) Completely Impatient User (CIU):

User who attempts to ISP_2 or goes to rest state RS in the $(n+1)^{th}$ attempt when was at ISP_1 in the n^{th}. Moreover, when was at ISP_2, moves to either ISP_1 or on RS in the next.

THEOREM 1: The n^th^ step transitions probability for FU to ISP_1 is

$$P\left[Y^{(2n)}=ISP_1\right]=pE^n \;;\; P\left[Y^{(2n+1)}=ISP_1\right]=0$$

where $B_1 = L_1(1-P_A)P_R$, $E = B_1 r$ and $n = 0,1,2,3$ - - -

PROOF: At $n=0$, $P\left[Y^{(0)}=ISP_1\right]=p$ and since, the transition over ISP_2 from ISP_1 is restricted, therefore following (11), the start of attempt is restricted to ISP_1 only $P\left[Y^{(0)}=RS\right]=0$

$$P\left[Y^{(1)}=ISP_1\right]=P\left[Y^{(0)}=RS\right]P\left[Y^{(1)}=ISP_1 \middle/ Y^{(0)}=RS\right]=0$$

$$P\left[Y^{(1)}=RS\right]=P\left[Y^{(0)}=ISP_1\right]P\left[Y^{(1)}=RS \middle/ Y^{(0)}=ISP_1\right]=pB_1$$

$$P\left[Y^{(2)}=ISP_1\right]=P\left[Y^{(1)}=RS\right]P\left[Y^{(2)}=ISP_1 \middle/ Y^{(1)}=RS\right]=pB_1 r$$

$$P\left[Y^{(2)}=RS\right]=P\left[Y^{(1)}=ISP_1\right]P\left[Y^{(2)}=RS \middle/ Y^{(1)}=ISP_1\right]=0$$

$$P\left[Y^{(3)}=ISP_1\right]=P\left[Y^{(2)}=RS\right]P\left[Y^{(3)}=ISP_1 \middle/ Y^{(2)}=RS\right]=0$$

$$P\left[Y^{(3)}=RS\right]=P\left[Y^{(2)}=ISP_1\right]P\left[Y^{(3)}=RS \middle/ Y^{(2)}=ISP_1\right]=pB_1^2 r$$

$$P\left[Y^{(4)}=ISP_1\right]=P\left[Y^{(3)}=RS\right]P\left[Y^{(4)}=ISP_1 \middle/ Y^{(3)}=RS\right]=pB_1^2 r^2$$

$$P\left[Y^{(4)}=RS\right]=P\left[Y^{(3)}=ISP_1\right]P\left[Y^{(4)}=RS \middle/ Y^{(3)}=ISP_1\right]=0$$

$$P\left[Y^{(5)}=ISP_1\right]=P\left[Y^{(4)}=RS\right]P\left[Y^{(5)}=ISP_1 \middle/ Y^{(4)}=RS\right]=0$$

$$P\left[Y^{(5)}=RS\right]=P\left[Y^{(4)}=ISP_1\right]P\left[Y^{(5)}=RS \middle/ Y^{(4)}=ISP_1\right]=pB_1^3 r^2$$

On continuing in similar way, the proof of the theorem exits.

THEOREM 2: The n^{th} step transitions probability for FU to ISP_2 is $P\left[Y^{(2n)}=ISP_2\right]=(1-p)D^n$;

$$P\left[Y^{(2n+1)}=ISP_2\right]=0$$

where $B_2 = L_2(1-P_A)P_R$, $D = B_2(1-r)$

THEOREM 3: For PIU the n^th^ step transition probability is:-

$$P\left[Y^{(2n)}=ISP_1\right]=pC^n \quad ;$$

$$P\left[Y^{(2n+1)}=ISP_1\right]=(1-p)A_2C^n$$

$$P\left[Y^{(2n)}=ISP_2\right]=(1-p)C^n \quad ;$$

$$P\left[Y^{(2n+1)}=ISP_2\right]=pA_1C^n$$

where $A_1 = L_1(1-P_A)(1-P_R)$, $A_2 = L_2(1-P_A)(1-P_R)$, $C = A_1 A_2$

THEOREM 4: For CIU, the n^{th} attempt approximate expressions are,

$$P\left[Y^{(2n)}=ISP_1\right]=p(C+E)^n \quad ;$$

$$P\left[Y^{(2n+1)}=ISP_1\right]=(1-p)A_2(C+D+E)^n$$

$$P\left[Y^{(2n)}=ISP_2\right]=(1-p)(C+D)^n \quad ;$$

$$P\left[Y^{(2n+1)}=ISP_2\right]=pA_1(C+D+E)^n$$

VI. SIMULATION BASED STATE PROBABILITY ANALYSIS

The expressions obtained in the theorem 1- 4 are examined through a graphical pattern for increasing number of attempts (n). Fig. 3 reflects that transition probability of ISP_1 varying over blocking probability L_1 and n. When L_1 increases, the chances of transition from ISP_1 also increases but the amount of this variation is very small.

For example, at n = 2, we get

$P\left[Y^{(2)}=ISP_1\right]$ = 0.124, when $L_1 = 0.3$,

$P\left[Y^{(2)}=ISP_1\right]$ = 0.204, when $L_1 = 0.5$

and $P\left[Y^{(2)}=ISP_1\right]$ = 0.364, when $L_1 = 0.8$.

While comparing ISP_1 with ISP_2 over n = 2,

$P\left[Y^{(2)}=ISP_1\right]$ = 0.124, when $L_1 = 0.5$ and

$P\left[Y^{(2)}=ISP_2\right]$ = 0.164, when $L_2 = 0.5$, one can find that with p = 0.8 for ISP_1, at n = 2 ISP_2 has better chance than ISP_1 (being L_1, L_2 equal). This is FU behavior.

With the increasing number of attempts the term $P\left[Y^{(n)}=ISP_1\right]=0$ as $n\rightarrow\infty$. This is interesting to observe that transition probabilities $P\left[Y^{(n)}=ISP_1\right]$ is zero in odd attempts (when n < 8). Fig. 4 is very similar to fig. 3 and supports the fact that the transition $P\left[Y^{(n)}=ISP_2\right]$ varies over increasing blocking probability for even attempts. This also constantly reduces over large n.

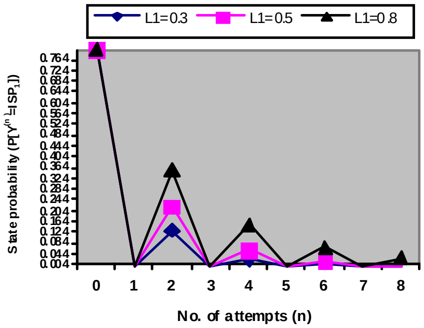

Fig. 3 For FU of operator ISP$_1$
(P$_A$ = 0.02, p = 0.8, P$_R$ = 0.8, r = 0.03)

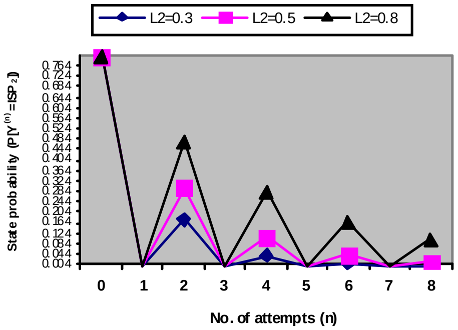

Fig. 4 For FU of operator ISP$_2$
(P$_A$ = 0.02, p = 0.8, P$_R$ = 0.8, r = 0.03)

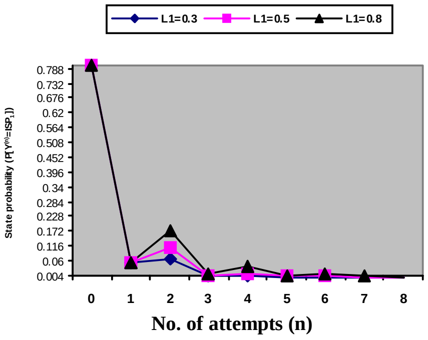

Fig. 5a For PIU of ISP$_1$
(P$_A$ = 0.02 , p = 0.8 , P$_R$ = 0.03 , L$_2$ = 0.3)

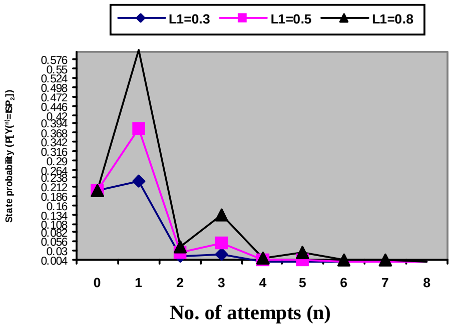

Fig. 5b For PIU of ISP$_2$
(P$_A$ = 0.02, p = 0.8 , P$_R$ = 0.03 , L$_2$ = 0.3)

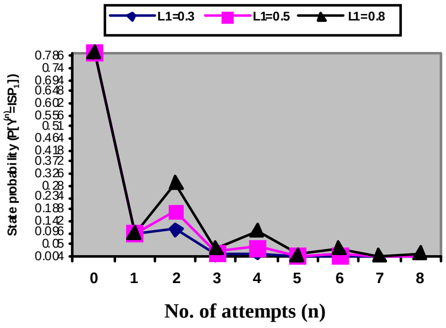

Fig. 5c For PIU of ISP$_1$
(P$_A$ = 0.02, p = 0.8, P$_R$ = 0.03, L$_2$ = 0.5)

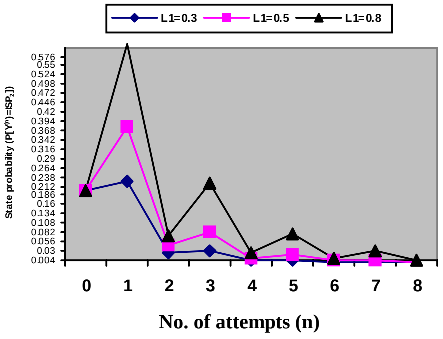

Fig. 5d For PIU of ISP$_2$
(P$_A$ = 0.02, p = 0.8, P$_R$ = 0.03, L$_2$ = 0.5)

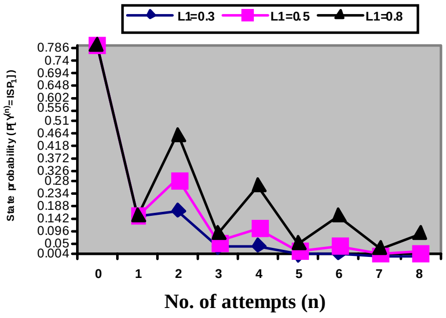

Fig. 5e For PIU of ISP$_1$
(P$_A$ = 0.02, p = 0.8, P$_R$ = 0.03, L$_2$ = 0.8)

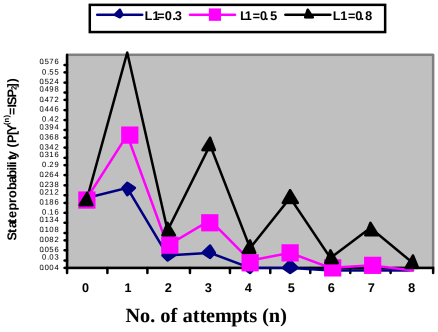

Fig. 5f For PIU of ISP$_2$
(P$_A$ = 0.02, p = 0.8, P$_R$ = 0.03, L$_2$ = 0.8)

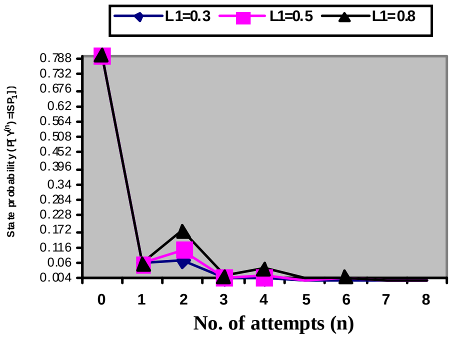

Fig. 6 a For CIU of ISP_1
($P_A = 0.02$, $p = 0.8$, $P_R = 0.03$, $L_2 = 0.3$)

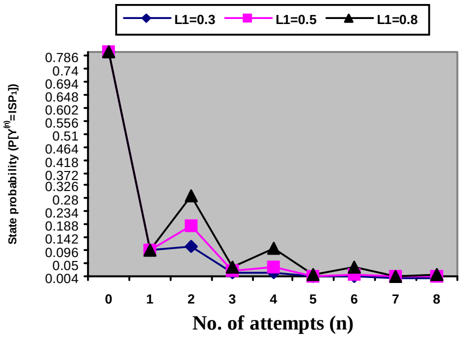

Fig. 6 c For CIU of ISP_1
($P_A = 0.02$, $p = 0.8$, $P_R = 0.03$, $L_2 = 0.5$)

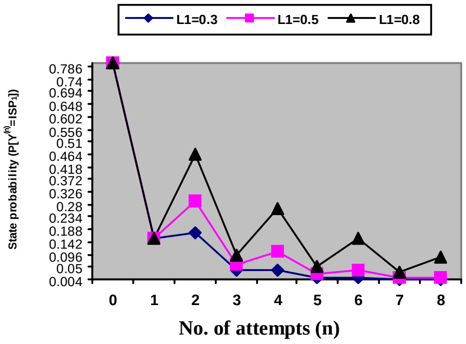

Fig. 6 e For CIU of ISP_1
($P_A = 0.02$, $p = 0.8$, $P_R = 0.03$, $L_2 = 0.8$)

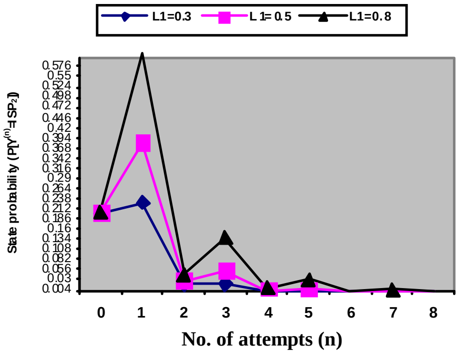

Fig. 6 b For CIU of ISP_2
($P_A = 0.02$, $p = 0.8$, $P_R = 0.03$, $L_2 = 0.3$)

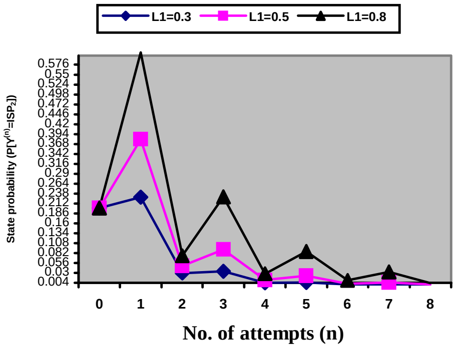

Fig. 6 d For CIU of ISP_2
($P_A = 0.02$, $p = 0.8$, $P_R = 0.03$, $L_2 = 0.5$)

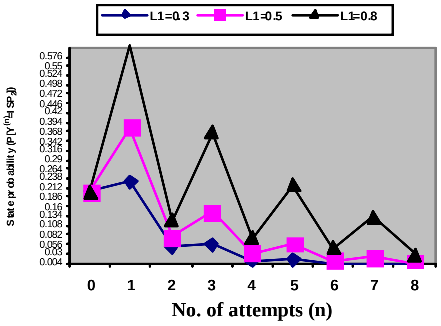

Fig. 6 f For CIU of ISP_2
($P_A = 0.02$, $p = 0.8$, $P_R = 0.03$, $L_2 = 0.8$)

According to fig. 5a, 5c, 5e, the competitor's blocking probability affects the PIU state probability because when L_1 and L_2 both are high; chances to ISP_1 are also high. From fig. 5b, 5d and 5f, one can find that for large number of attempts, the transition probability reaches to zero with the joint condition of large L_1 and L_2. At n = 1, the highest transition probability found, followed by next highest at n = 3 for ISP_2, but this highest amount decreases with increasing L_2.

In another comparison, when n = 2 and n = 4, L_1, L_2, kept fixed, we have

$$P\left[Y^{(2)} = ISP_1\right] = 0.142, \text{ when } L_1 = 0.5, L_2 = 0.5,$$

$$P\left[Y^{(2)} = ISP_2\right] = 0.212, \text{ when } L_1 = 0.5, L_2 = 0.5$$

$$P\left[Y^{(4)} = ISP_1\right] = 0.004, \text{ when } L_1 = 0.5, L_2 = 0.5$$

$$P\left[Y^{(4)} = ISP_2\right] = 0.056, \text{ when } L_1 = 0.5, L_2 = 0.5.$$

In second attempt, the chance for ISP_2 are high and continues for fourth attempt also, but this difference reduces over increasing n. PIU prefers to ISP_2 more up to fourth attempt even when p = 0.8 for ISP_1 exists. This is PIU behavior.

In light of fig. 6a, 6c, and 6e, the same pattern found as discussed for PIU. The increase in L_2 constantly produces significant increase in transitions for increasing L_1. Similar happens in fig. 6b, 6d and 6f. For small opponent's blocking leads to less number of attempts in order to reach the transition probability equal to zero. More and more attempts are needed to stabilize the transition process if L_1 and L_2 both are towards higher side. So, behavior of CIU are same as PIU in terms of state probabilities.

I. CONCLUDING REMARKS

State probabilities depend on number of call attempts made by user for getting Internet connected. This probability reduces sharply as attempt increases. The FU users have a tendency to stick with their favourate operators up to seven to eight attempts but PIU group has negative tendency in this regards. In contrary, CIU users bear a better proportion of state probabilities. When blocking of the network of ISP_1 is high then he gains state probabilities related to FU users. Moreover, the increase of L_2 provides gain in terms of higher proportion of traffic of ISP_1.

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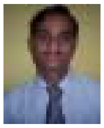
Author's Biography

Dr. Diwakar Shukla is working as an Associate Professor in the Department of Mathematics and Statistics, Sagar University, Sagar, M.P. and having over 20 years experience of teaching to U.G. and P.G. classes. He obtained M.Sc.(Stat.), Ph.D.(Stat.), degree from Banaras Hindu University, Varanasi and served the Devi Ahilya University, Indore, M.P. as a Lecturer over nine years and obtained the degree of M. Tech. (Computer Science) from there. During Ph.D., he was junior and senior research fellow of CSIR, New Delhi qualifying through Fellowship Examination (NET) of 1983. Till now, he has published more than 55 research papers in national and international journals and participated in more than 35 seminars / conferences at national level. He is the recipient of MPCOST Young Scientist Award, ISAS Young Scientist Medal, UGC Career Award and UGC visiting fellow to Amerawati University, Maharashtra. He also worked as a selected Professor to the Lucknow University, Lucknow, U.P., for one year and visited abroad to Sydney (Australia) and Shanghai (China) for conference participation. He has supervised seven Ph.D. theses in Statistics and Computer Science both; and eight students are presently enrolled for their doctoral degree under his supervision. He is member of 10 learned bodies of Statistics and Computer Science both at national level. The area of research he works for are Sampling Theory, Graph Theory, Stochastic Modelling, Computer Network and Operating Systems.



Dr. Sanjay Thakur has completed M.C.A. and Ph.D. (CS) degree from H.S. Gour University, Sagar in 2002 and 2009 respectively. He is presently working as a Lecturer in the Department of Computer Science & Applications in the same University since 2007. He did his doctoral work in the field of Computer Networking and Internet traffic sharing. He has authored and co-authored 10 research papers in National/International journals and conference proceedings. His current research interest is Stochastic Modeling of Switching System of Computer Network and Internet Traffic Sharing Analysis.

Arvind Kumar Deshmukh has completed M.Sc.(Physics) inYear 2000 and MCA Degree in Year 2003 from Dr. Hari Singh Gour University Sagar, MP, India. He is presently working as a Lecturer in the Department of Computer Science & Applications in the same University since 2004. He is pursuing his Ph.D. work as a registered scholar.